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LUNAR SURFACE ENGINEERING PROPERTIES EXPERIMENT DEFINITION

FINAL REPORT: VOLUME IV OF IV
FLUID CONDUCTIVITY OF LUNAR SURFACE MATERIALS

by

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D. ROGER WILLIS

PREPARED FOR MARSHALL SPACE FLIGHT CENTER
HUNTSVILLE, ALABAMA UNDER NASA CONTRACT
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PREFACE

This report presents the results of studies conducted during the period June 20, 1968 - July 19, 1969, under NASA research contract NAS 8-21432, "Lunar Surface Engineering Properties Experiment Definition." This study was sponsored by the Advanced Lunar Missions Directorate, NASA Headquarters, and was under the technical cognizance of Dr. N. C. Costes, Space Science Laboratory, George C. Marshall Space Flight Center.

The report reflects the combined effort of four faculty investigators, a research engineer, a project manager, and six graduate research assistants, representing several engineering and scientific disciplines pertinent to the study of lunar surface material properties. James K. Mitchell, Professor of Civil Engineering, served as Principal Investigator and was responsible for those phases of the work concerned with problems relating to the engineering properties of lunar soils and lunar soil mechanics. Co-investigators were William N. Houston, Assistant Professor of Civil Engineering, who was concerned with problems relating to the engineering properties of lunar soils; Richard E. Goodman, Associate Professor of Geological Engineering, who was concerned with the engineering geology and rock mechanics aspects of the lunar surface; and Paul A. Witherspoon, Professor of Geological Engineering, who conducted studies related to thermal and permeability measurements on the lunar surface. Dr. Karel Drozd, Assistant Research Engineer, performed laboratory tests and analyses pertinent to the development of a borehole probe for determination of the in-situ characteristics of lunar soils and rocks. John Hovland, David Katz, Laith I. Namiq, James B. Thompson, Tran K. Van, and Ted S. Vinson served as Graduate Research Assistants and carried out many of the studies leading to the results presented in this report. Francois Heuzé, Assistant Specialist, served as project manager and contributed to studies concerned with lunar rock mechanics.

Ultimate objectives of this project are:

1) Assessment of lunar soil and rock property data using information obtained from Lunar Orbiter and Surveyor missions.
2) Recommendation of both simple and sophisticated in-situ testing techniques that would allow determination of engineering properties of lunar surface materials.
3) Determination of the influence of variations in lunar surface conditions on the performance parameters of a lunar roving vehicle.
4) Development of simple means for determining the fluid and thermal conductivity properties of lunar surface materials.
5) Development of stabilization techniques for use in loose, unconsolidated lunar surface materials to improve the performance of such materials in lunar engineering application.

The scope of specific studies conducted in satisfaction of these objectives is indicated by the following list of contents from the Detailed Final Report which is presented in four volumes. The names of the investigators associated with each phase of the work are indicated.

VOLUME I

MECHANICS AND STABILIZATION OF LUNAR SOILS

1. Lunar Soil Simulation
(W. N. Houston, L. I. Namiq, and J. K. Mitchell)

2. Lunar Surface Trafficability Studies
(J. B. Thompson and J. K. Mitchell)

3. Foamed Plastic Chemical Systems for Lunar Soil Stabilization Applications
(T. S. Vinson and J. K. Mitchell)

VOLUME II

LUNAR SOIL PROPERTIES FROM PHOTOGRAPHIC RECORDS

VOLUME III

BOREHOLE PROBES

VOLUME IV

FLUID CONDUCTIVITY OF LUNAR SURFACE MATERIALS

VOLUME IV OF FOUR

CONTENTS

CHAPTER 1

STUDIES ON CONDUCTIVITY OF LUNAR SURFACE MATERIALS

(D. F. Katz, P. A. Witherspoon, and D. R. Willis)

I. OBJECTIVES

The overall objective of this investigation is to develop a means of measuring the permeability of lunar soils and rocks in situ. It has been proposed to design and test a surface probe that can measure permeabilities with reasonable accuracy. Because of the lack of any atmosphere on the moon, it will be necessary to utilize gas in operating the probe. In the current year's work, a theory of gas flow in porous media appropriate to the surface probe has been developed. The theory is applicable to probe operation in the lunar environment as well as on earth. In addition, the design of experiments to check the theory has begun.

II. INTRODUCTION

A. Conceptual Description of Probe

A schematic drawing of the probe is given in Figure 1-1. The system basically consists of a holding chamber containing pressurized fluid, a disc-shaped source, and pressure measuring devices imbedded along with the source in a circular, impermeable skirt. The holding chamber contains the charge of gas for an individual measurement, and is connected by valves to the disc source and to a larger gas storage tank. A single switch releases the charge of gas, and activates a timer linked to the surface pressure tap. Constant source pressure is achieved by a servo connection between the release valve and an exit tap. The connecting valve between the holding

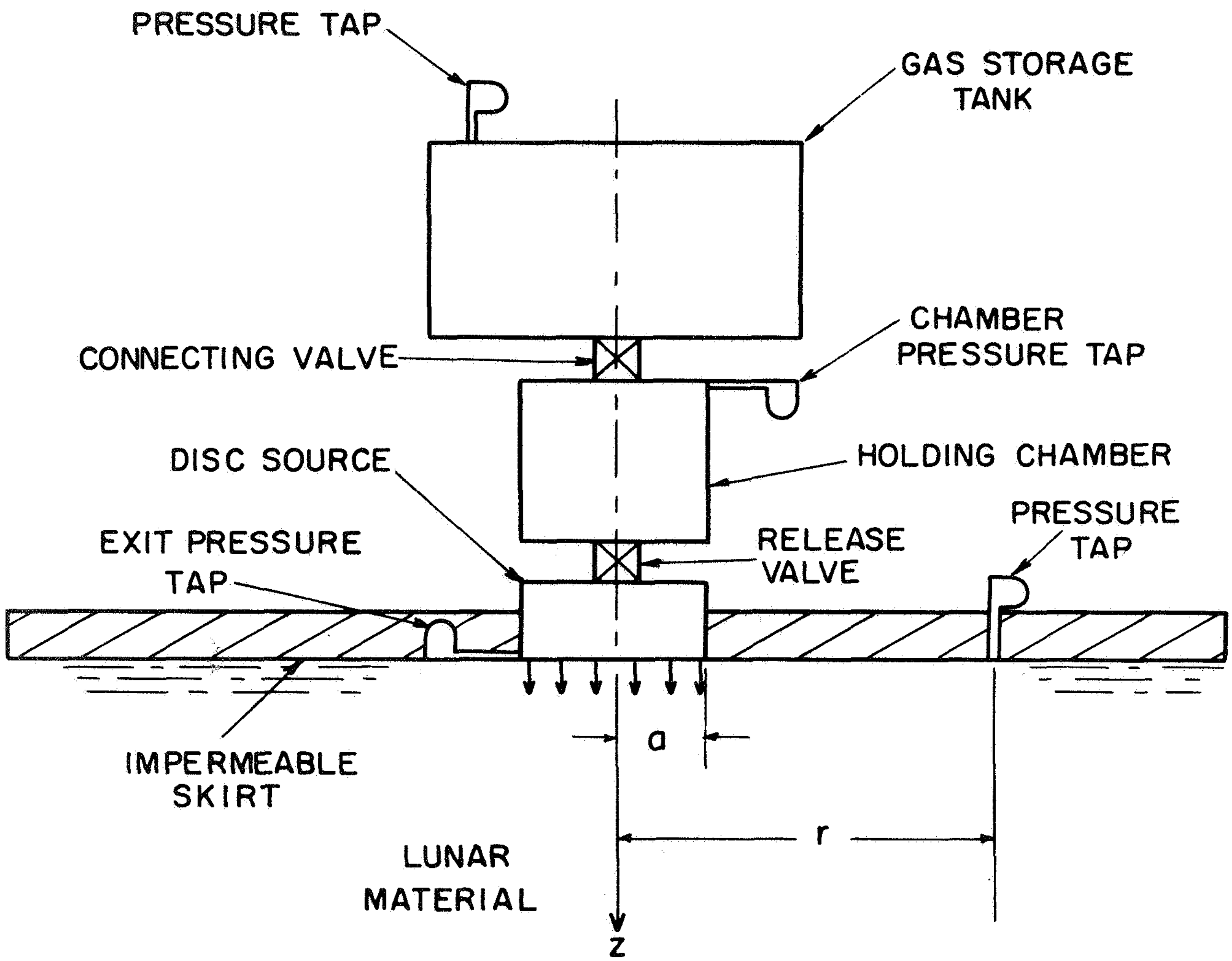

FIGURE 1-1 PERMEABILITY PROBE

chamber and the storage tank insures that only a small portion of the gas supply is consumed per measurement. This also allows for variation in source pressure as required by varying permeability. One can then compute permeability from a record of pressures and flow rates.

In the design of the probe, due consideration must be given to size, weight, durability requirements, etc. Conceptually, it seems feasible to utilize a waste gas, such as CO_2. In view of current practice on earth, a miniaturization of the probe is suggested. However, any miniaturization is limited by the requirement that the source diameter be large compared to the pore dimensions.

B. Flow in Porous Media in the Lunar Environment

Due to the high vacuum in the lunar environment, there exists a basic problem of determining the dominant flow regime during probe operation. As the average pore size and/or fluid pressure decrease, the fundamental nature of gas flow changes. The initial departure from viscous flow is the relaxation of the no slip boundary condition on the interstitial surfaces. The resulting, augmented flow, while still viscous in nature, exhibits a greater temperature dependence than previously. As the effective degree of rarefaction increases further, the continuum nature of the fluid breaks down, and the flow must be modeled from a molecular point of view. In general, account must be taken of both intermolecular collisions, and collisions between molecules and the interstitial boundaries. However, when a high degree of rarefaction is achieved, the former become negligible due to the relative scarcity of fluid molecules. The fluid parameter

indicative of the particular flow regime prevalent is the Knudsen number, defined here as $Kn_d \equiv \frac{\lambda}{d}$, where λ is the mean free path of the molecules of the fluid, and d is an average pore cross sectional dimension. For viscous flow, $Kn_d \ll 1$, whereas for the so-called Knudsen flow, $Kn_d \gg 1$.

The geometrical complexity of the pore structure of rocks precludes "exact" solutions of the fluid equations of motion, and one is forced to work in terms of averaged flow quantities; see Appendix A. This approach has proven successful in treating terrestrial, continuum flows. It is, therefore, not unreasonable to expect that transitional flow in the lunar environment may also be amenable to such an approach. Henceforth, all flow quantities referred to are averages. It is also necessary to assume that the porous medium of interest is homogeneous and isotropic.

It seems likely that in the immediate neighborhood of the fluid source, the flow will be viscous in nature, and, thus, basically predictable by current techniques. However, the nature of the evolution of Knudsen flow, as the distance from the source increases, is extremely difficult to foresee at this time. Detailed theories for transition flow, namely solutions of the Boltzmann equation, for even the simplest geometries are quite scarce. As a result, this report introduces the concept of local similarity in treating the gas flow. This approach makes maximum use of physical intuition about the flow field, and requires experimental calibration of the probe on a sample of known material. It does not, however, require actual solution of the fluid equations of motion. Such an approach to the problem seems far more reasonable, as a first effort, than some approximate over-simplified solution.

III. THE LOCAL SIMILARITY METHOD

A. Basic Theory

In the analysis of terrestrial fluid flow in porous media, effective use if made of a simplified, integrated form of the equation of motion in which the velocity, or flow rate, is taken to be proportional to the pressure gradient. For continuum flow this is known as Darcy's Law, and can be written

$$\underline{v} = C(p)\nabla p \qquad (1\text{-}1)$$

where $\underline{v}$ is the velocity vector and p is pressure. The coefficient C contains parameters that characterize the particular problem, basically viscosity and appropriate pore dimensions. For incompressible, continuum flow, C is independent of pressure, and is normally written

$$C \equiv \frac{k}{\mu} \qquad (1\text{-}2)$$

where μ is viscosity, and k is termed the permeability. Thus, permeability depends upon the size and shape of the interstices, and has dimensions of length squared. For both slip flow and Knudsen flow, C is pressure dependent (Carmen, 1956). It is, therefore, suggested that Equation (1-1) may be characteristic of flow in porous media in general. This assumption is made, and thus C(p) can be thought of as a "master" diffusion coefficient encompassing all flow regimes. C(p) can be rendered dimensionless by defining

$$\tilde{C}(p) \equiv \frac{\mu}{L^2} C(p) \tag{1-3}$$

where $\tilde{C}$ is a dimensionless function of pressure. It is assumed that $\tilde{C}$ is a function of local pressure (or Knudsen number)* only. L^2 has dimensions of area, and can be thought of as a generalized permeability, applicable to all flow regimes. These are the basic local similarity assumptions. They embody all the necessary physics of the flow field. The validity of these assumptions will be discussed below.

Consider now a steady, isothermal flow field dependent upon only one space variable, denoted by r, i.e. a one-dimensional, cylindrically symmetric, or spherically symmetric problem. Henceforth such problems will be referred to as "symmetric." Equation (1-1) becomes

$$v = C(p)\frac{dp}{dr} \tag{1-4}$$

The continuity equation can be written

$$\alpha \rho v r^j = \hat{Q} \tag{1-5}$$

where α is the area fraction, cf. Appendix A ; ρ is the average density and $\hat{Q}$ takes on different values (see Table 1-1), depending upon the nature of the symmetric problem.

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*Pressure and Knudsen number are inversely proportional. See Appendix B .

TABLE 1-1

Symbol	Type of Symmetric Problem		
	One-Dimensional	Cylindrical	Spherical
j	0	1	2
$\hat{Q}$	Q/A_r	$Q/2\pi\ell_c$	$Q/4\pi$

Here Q refers to the dimensional mass flow rate, A_r, is the total cross-sectional area of the rock in one-dimensional flow, and ℓ_c is the cylinder length for cylindrically symmetric flow. Combining Equations (1-3), (1-4), and (1-5).

$$\hat{Q} = \alpha \frac{L^2}{\mu} \rho r^j \tilde{C}(p) \frac{dp}{dr} \tag{1-6}$$

Assume that the gas is perfect, so that $p = \rho RT$, where R is the gas constant and T is the temperature. Then

$$\hat{Q} = \alpha \frac{L^2}{\mu RT} r^j p\tilde{C}(p) \frac{dp}{dr} \tag{1-7}$$

Since $\tilde{C}(p)$ is dimensionless, it can be expressed as a function of a dimensionless pressure ζ:

$$\tilde{C}(p) \equiv \frac{1}{F(\zeta)} \tag{1-8}$$

The particular form of ζ is motivated by the fact that $\tilde{C}$ can be considered as being only a function of an appropriate Knudsen number (see Appendix B). Thus

$$\zeta \equiv \frac{L\overline{V}}{2\mu RT}\, p = \frac{1}{Kn_L} \qquad (1\text{-}9)$$

where $\overline{V} = \sqrt{\frac{8RT}{\pi}}$, the mean thermal speed, and Kn_L is the Knudsen number based on the "length" L, $Kn_L \equiv \frac{\lambda}{L}$. Substituting in Equation (1-7) and rearranging,

$$\alpha L^2 \frac{1}{\hat{Q}\mu RT}\, r^j p\, \frac{dp}{dr} = F(\zeta) \qquad (1\text{-}10)$$

This is a fundamental similarity relation. $F(\zeta)$ is a "universal" function, in that data from all symmetric problems, plotted according to Equation (1-10) fall on the same curve. It should be noted that $F(\zeta)$ is not the only possible universal curve. Multiplication by any real function of ζ yields an equally universal curve.

B. Method for Determining Permeability and Area Fraction

By developing dimensionless relationships in this manner, Equation (1-10) can be used to determine both permeability L^2 and the area fraction α. For example, if the flow field for the lunar probe can be approximated as spherically symmetric, then $j = 2$ and $\hat{Q} = Q/4\pi$. Equation (1-10) becomes

$$\alpha L^2 \left(\frac{4\pi}{Q\mu RT}\, r^2 p\, \frac{dp}{dr} \right) = F(\zeta) \qquad (1\text{-}11)$$

Since $F(\zeta)$ presumably holds for any symmetric flow in porous media, this function can be uniquely determined by appropriate experimentation in the laboratory.

To apply Equation (1-11) to an unknown rock sample, one must fix the flow rate Q and measure temperature T, from which $\mu = \mu(T)$ is easily determined. If pressure taps are appropriately spaced in the skirt of the probe (cf. Figure 1-1) then $r^2 p \frac{dp}{dr}$ can be measured at two or more different values of r. This enables one to determine two different values of the bracketed expression in Equation (1-11) which are sufficient to determine both L and α.

A simple procedure illustrating the method of calculation can be outlined as follows. From Equations (1-9) through (1-11)

$$\ln \zeta = \ln p + \ln\left(\frac{\bar{V}}{2\mu RT} L\right) \tag{1-12a}$$

$$\ln F(\zeta) = \ln\left(r^2 p \frac{dp}{dr}\right) + \ln\left(\alpha L^2 \frac{4\pi}{Q\mu RT}\right) \tag{1-12b}$$

$$\frac{d \ln F(\zeta)}{d \ln \zeta} = \frac{d \ln\left(r^2 p \frac{dp}{dr}\right)}{d \ln p} \tag{1-12c}$$

If follows that curves of $F(\zeta)$ vs ζ and $r^2 p \frac{dp}{dr}$ vs p, plotted on identical log-log scales, differ only in the positions of their respective origins of coordinates. Thus, the two measured values of $r^2 p \frac{dp}{dr}$ are plotted versus p on log-log paper, and $F(\zeta)$ vs ζ is plotted on identical paper. The two plots are then placed on top of each other, and maneuvered, keeping respective axes mutually parallel, until

the two experimental points lie on the known curve. The coordinates of the origin of the experimental plot relative to the universal one then clearly yield both α and L. See Figure 1-2.

As drawn in Figure 1-2, $F(\zeta)$ will be a monatonic function of ζ. However, a problem of uniqueness in determining α and L will arise if there are regions where, on the log-log scale, $F(\zeta)$ is locally linear. If the experimental points correspond to such regions, then the position of the origin of the experimental plot is not uniquely determined. This difficulty can, principle, be circumvented, however, by redefinition of the universal curve such that it has adequate curvature throughout.

C. Discussion of Assumptions

The concept of local similarity contains the following basic assumptions:

(1) flow is steady

(2) flow is independent of the initial pressure in the porous medium

(3) a single length characterizes the fluid conductivity of the porous medium

(4) flow is isothermal.

The first and second assumptions are not independent. This is shown qualitatively in Figure 1-3. Clearly the flow field in the entire rock is not steady, due to the presence of the pressure (and density) "wavefront," i.e. the boundary between those portions of the rock

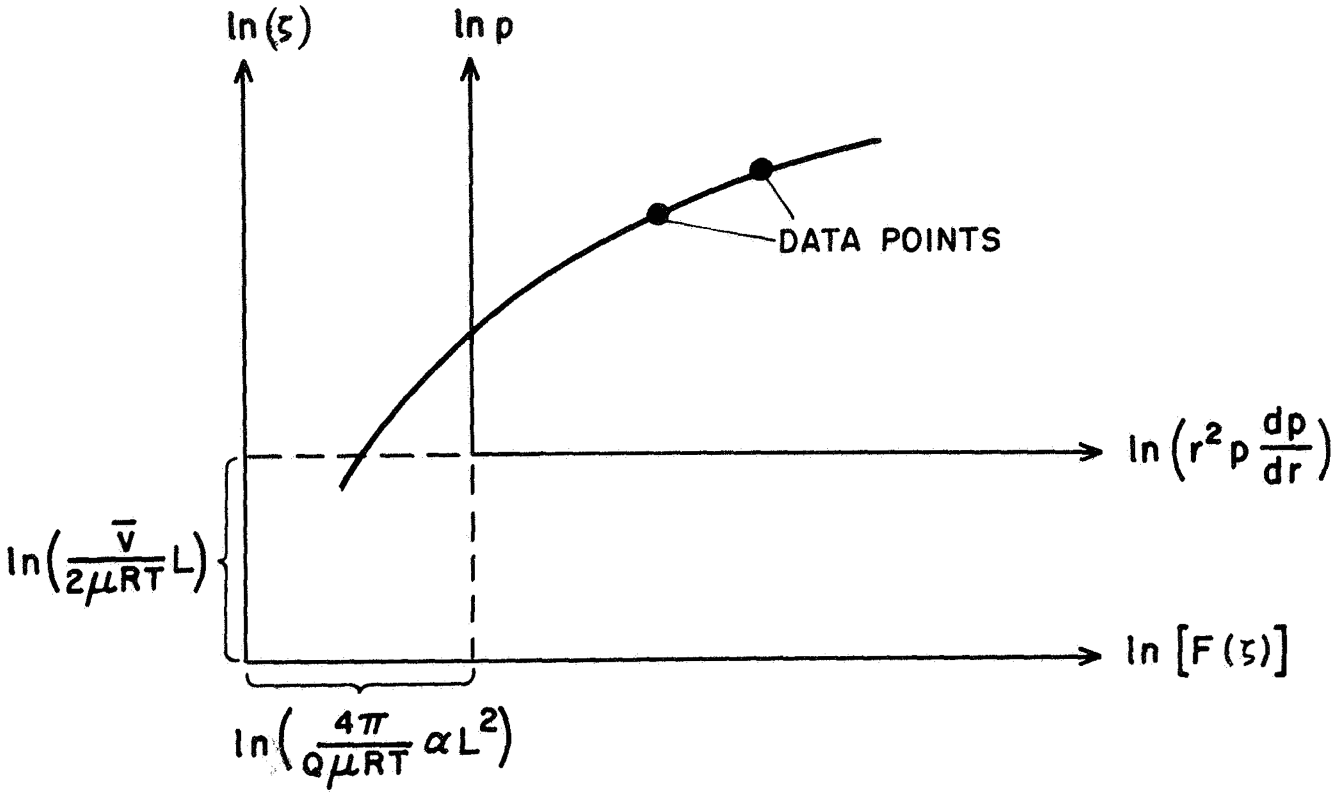


FIGURE 1-2 EXAMPLE OF MATCHING PROCEDURE IN DETERMINING PERMEABILITY AND AREA FRACTION

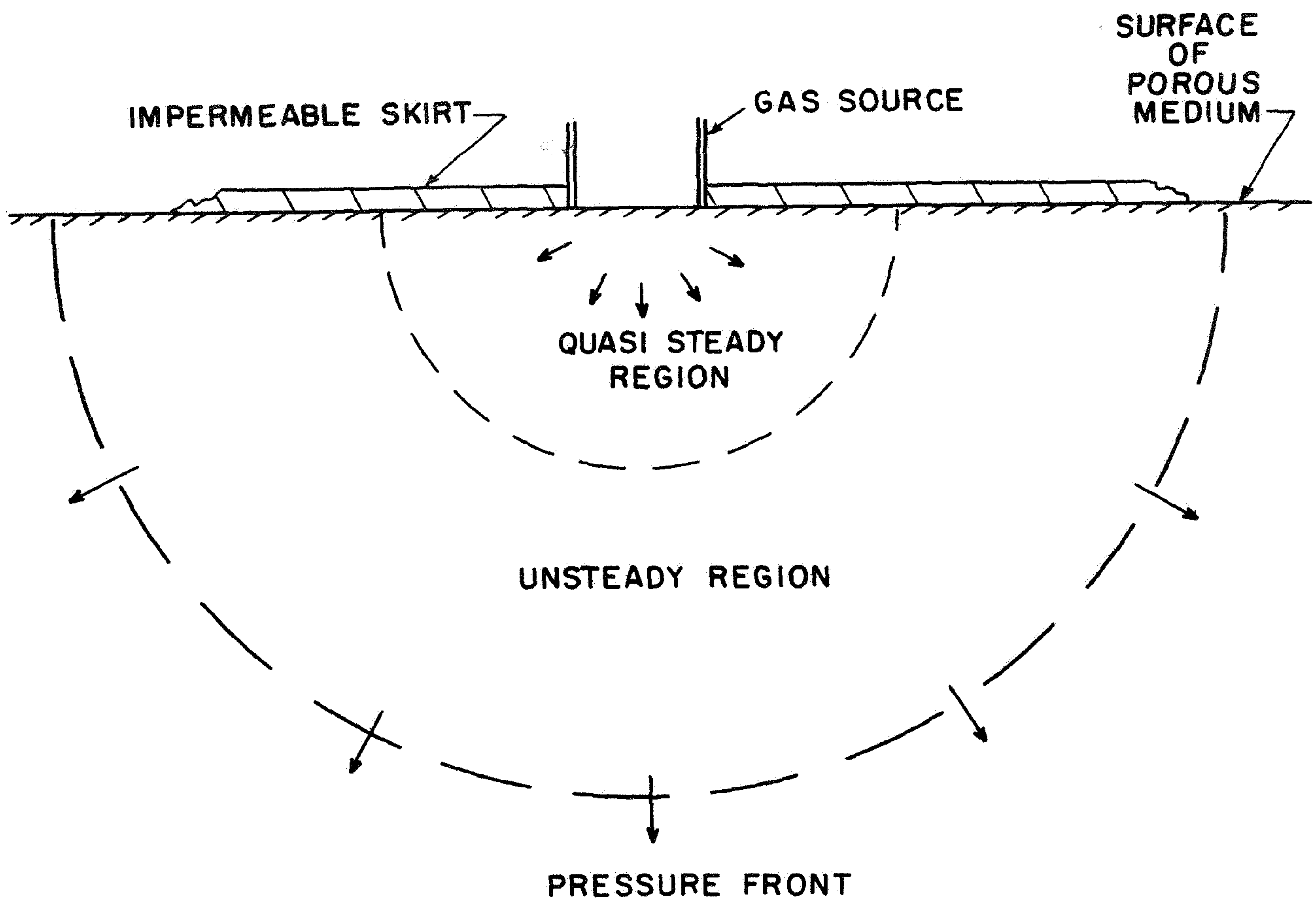


FIGURE 1-3 QUALITATIVE TIME DEPENDENCE OF FLOW FIELD FOR SURFACE PROBE

that contain gas molecules from the source, and those that do not. The velocity of propagation of this front will depend in part upon the initial fluid pressure in the rock. Sufficiently close to the source, however, it is not unreasonable to presume that time variations in the flow field will be less significant, and subsequently, the influence of the initial pressure. Strictly speaking, then, the local similarity theory can only be applicable in this "quasi steady" region. It is assumed that this region is large enough to permit the necessary pressure measurements.

The assumption of steady flow for operation of the surface probe can be examined quantitatively for two limiting cases: (a) slightly compressible, continuum flow, and (b) Knudsen flow. In both cases the governing equation becomes the diffusion equation (Carman, 1956). A solution to the equation for the precise geometry of the probe (without the assumption of spherically symmetric flow) has been given by Selim et al. (1963). Steady state flow is effectively reached for $\tau = 10^3$, where τ is a dimensionless time which can be written $\tau = \frac{4Dt}{a^2}$. Here t is time, "a" is the source radius, and D can be thought of as the diffusivity appropriate to the particular flow regime. For slightly compressible, continuum flow, $D \equiv \frac{kp}{\phi\mu}$, where ϕ is porosity and k is the continuum permeability mentioned earlier. For Knudsen flow, $D = \frac{1}{3}\frac{d\bar{v}}{\phi}$, i.e., D is simply the Knudsen diffusivity D_{KA} divided by porosity.

For continuum flow, consider a typical situation where $\phi \simeq 10^{-1}$, $a \simeq 10^{-1}$ cm, $\mu \simeq 10^{-1}$ poise, $p \simeq 10^6$ dyne/cm^2, and 10^{-11} cm$^2 \lesssim k \lesssim 10^{-8}$ cm^2. (Continuum flow isn't expected for $k \lesssim 10^{-11}$ cm^2.) Then 10^{-2} sec $\lesssim t \lesssim 10$ sec, so that steady state flow is feasible.

Now consider a typical Knudsen flow situation with the same values of ϕ and "a" as above. Let $T \simeq 10^2$ °K so that $V \simeq 10^4$ cm/sec. For most rocks, 10^{-6} cm $\lesssim d \lesssim 10^{-3}$ cm. It follows that $10^{-2} \lesssim t \lesssim 10^2$ sec. So, for the rarefied limit, as well as the continuum one, steady state flow appears feasible in probe operation.

It is also instructive to utilize the Selim et al. solution in considering the assumption of spherically symmetric flow. Their results show that, at the surface of the porous medium, for $r \leq 5.633a$ the steady flow field appears spherically symmetric. Thus, the surface pressure taps in the probe would have to be replaced in this region, a quite reasonable design requirement. The Selim et al. results also show that over practical time periods, there is essentially no flow at the rock surface for $r \leq (10a)$. Thus, for $a = 0.1$ cm, a reasonable radius for the impermeable skirt might be, say, 3 cm. This requirement is also compatible with the envisioned probe design.

The problem of characteristic lengths is fundamental to flow in porous media, and, even for purely continuum flows, is as yet unresolved. In the local similarity analysis there can be formal dependence upon only a single characteristic dimension, L. Current theory on flow in porous media often introduces two characteristic dimensions. One is associated with effective pore diameter (denoted above by d), and the other, with effective pore length. If the proposed experimentation reveals the necessity of introducing a second characteristic dimension, it will have to be done empirically.

IV. COMPARISON WITH EXISTING THEORY

The most systematic approach to the problem of transitional flow in porous media has been made by Mason et al. (1961, 1962, 1963, 1964, 1967) in the form of the "dusty gas models". This model treats the overall transport of a single or multicomponent gas in a porous medium as a multiple diffusion problem. The pore structure is considered to be one of the diffusing species, but is constrained to remain stationary as a uniform distribution of "dust" particles. The starting point in the analysis is a "master" diffusion equation governing the effects of viscous transfer of momentum in the diffusion of a multicomponent gas mixture. This equation was derived using the advanced "thirteen moment" approximation of the kinetic theory of gases (Zhdanov, 1962). Only steady one-dimensional flows are considered, and inertial effects in the continuum regime are neglected. Porosity and tortuosity effects are introduced empirically. For isothermal flow of a single component gas, the following equation is derived:

$$Q = \left\{a_1 p + b\left(\frac{1 + c_1 p}{1 + c_2 p}\right)\right\}\frac{dp}{dr} \tag{1-13}$$

Here a_1, b, c_1, c_2 are constants, one of which must be determined experimentally. Since $Q = \rho r \alpha A$, where A is the area of this fluid source, Equation (1-13) can be rewritten:

$$v = \frac{RT}{\alpha A}\left\{\frac{1}{p}\left(a_1 p + b\left(\frac{1 + c_1 p}{1 + c_2 p}\right)\right)\right\}\frac{dp}{dr} \tag{1-14}$$

Equation (1-14) is consistent with the local similarity expression, Equation (1-4), for one dimensional flow with

$$C(p) = \frac{RT}{\alpha A}\left[\frac{1}{p}\left(a_1 p + b\,\frac{1 + c_1 p}{1 + c_2 p}\right)\right] \qquad (1\text{-}15)$$

Equation (1-15) has the same quantitative pressure dependence as an equation obtained by Wakao et al. (1965). For steady, one-dimensional isothermal flow that is undergoing transition in porous media. The result of Wakao et al. can be expressed as

$$C(p) = \frac{1}{p}\left[\frac{\frac{r^2 p_o}{8\mu}\left(\frac{p}{p_o}\right)^2}{\frac{p}{p_o} + Kn_{d_o}} + \frac{\left(\frac{\pi}{4}\right) D_{KA}\,\frac{1}{Kn_{d_o}}\,\frac{p}{p_o}}{1 + \frac{1}{Kn_{d_o}}\,\frac{p}{p_o}} + \frac{D_{KA}}{1 + \frac{1}{Kn_{d_o}}\,\frac{p}{p_o}}\right] \qquad (1\text{-}16)$$

The subscript o refers to reference conditions at the gas source. The first term in Equation (1-16) corresponds to simple Poiseuille flow. Wakao's second term, to slip flow, and the third term, to Knudsen flow. Wakao's equation contains no disposable constants, but its justification is largely heuristic, and is based on the single capillary model for flow in porous media. Similarly, the additional efforts, listed in Reference 8, are essentially heuristic. Unfortunately, experimental verification for all the above work has been confined to measurements on single capillaries or bundles of capillaries.

The only experimentation on transitional flows in actual porous media has been the recent work of Huang and Ramsey (1968). Their experiments consisted of steady, one-dimensional flows, in which large relative pressure differences between the ends of the rock samples were achieved. However, all pressures remained of the order of atmospheric, and rarefied effects were assumed to be present only by virtue of the small pore

sizes of the rock samples. The data are in good agreement with the Wakao equation, cf. Equation (1-16). However, it should be emphasized that the applicability of the data to the present problem is questionable due to the lack of in-vacuo conditions.

The final work of interest is that of Sreekanth (1968). He conducted experiments on transitional flows in single tubes in which large relative pressure drops were achieved, the tube exits being maintained at vacuum conditions. Three different tubes were used, with length-to-diameter ratios of 4.77, 9.66, and 15.03. The flows were steady and isothermal. Sreekanth's data for pressure distribution agree extremely well with a continuum analysis, utilizing an approximate method of accounting for compressibility and assuming velocity slip (Ebert and Sparrow, 1965). Such an analysis is consistent with the local similarity approach, and an effective C(p) can be deducted.

$$C(p) = \frac{4RT}{p_o{}^2}\frac{Q}{d^2}\left\{\left(\frac{p_o}{p}\right)^3\left(\frac{b_1 \mathrm{ReKn}_{d_o}\chi}{4}\right) + \left(\frac{p_o}{p}\right)^2\left(\frac{\mathrm{Re}\chi}{2}\right) - \left(\frac{p_o}{p}\right)\left(\frac{\pi b_1}{8\mathrm{ReKn}_{d_o}}\right) - \frac{\pi}{64\mathrm{ReKn}_{d_o}^2}\right\} \tag{1-17}$$

Here $\mathrm{Re} \equiv \frac{4}{\pi}\frac{Q}{\mu d}$, a Reynolds number based on mass flow rate; b_1 is a constant, of order one, contained in the slip boundary condition $(v)_{wall} = -b_1\lambda\left(\frac{\partial v}{\partial n}\right)_{wall}$, where $\left(\frac{\partial v}{\partial n}\right)_{wall}$ is the normal derivative of velocity at the wall; and χ is a weakly varying function of local Knudsen number, which is effectively a constant of order one.

Due to the fact that the Wakao et al. approach, cf. Equation (1-16). which is intended for flow in porous media, is derived from a capillary model, it is instructive to apply their equation to the Sreekanth data. In so doing, it is necessary to select only those results with small values of Re and large values of $\nabla p/p_e$ because these are appropriate to transitional flow in porous media. Here ∇p is the pressure drop along the tube, and p_e is the background pressure. If calculated volumetric flow rates $\dot{V}$ are compared with the experimental data, the accuracy improves with increasing length-to-diameter ratio. This is to be expected since the Wakao theory assumes fully developed flow, independent of capillary entrance effects. Typical results for the longest of the three tubes are shown in Table 1-2. As expected, the

TABLE 1-2

COMPARISON OF VOLUMETRIC FLOW RATES
CALCULATED FROM WAKAO THEORY WITH SREEKANTH DATA

$\frac{\Delta p}{p_e}$	$\dot{V}_{calculated}\left(\frac{cm^3}{min}\right)$ (Wakao et al.)	$\dot{V}_{measured}\left(\frac{cm^3}{min}\right)$ (Sreekanth)	Percentage error %
1.45	78.76	71.64	9
2.84	639.52	556.00	13.1
7.86	1199.54	990.50	17.4
10.25	335.64	293.00	12.7
20.05	31.254	29.48	5.6
26.53	6.02	5.95	1.2

greatest accuracy is for the highest $\frac{\Delta p}{p_e}$, and lowest $\dot{V}$ (i.e. the lowest Reynolds number).

However, the calculated pressures along the tube length do not compare nearly as favorably with the data for large values of $\frac{\Delta p}{p_e}$. In some instances, these pressures become double-valued near the tube end. Typical results for the longest tube are shown in Figure 1-4. Clearly, then, the approaches of Sreekanth and Wakao et al. are not equivalent. Nonetheless, while there is some question as to which is more applicable to flow in porous media, neither contradicts the local similarity hypothesis made in this work.

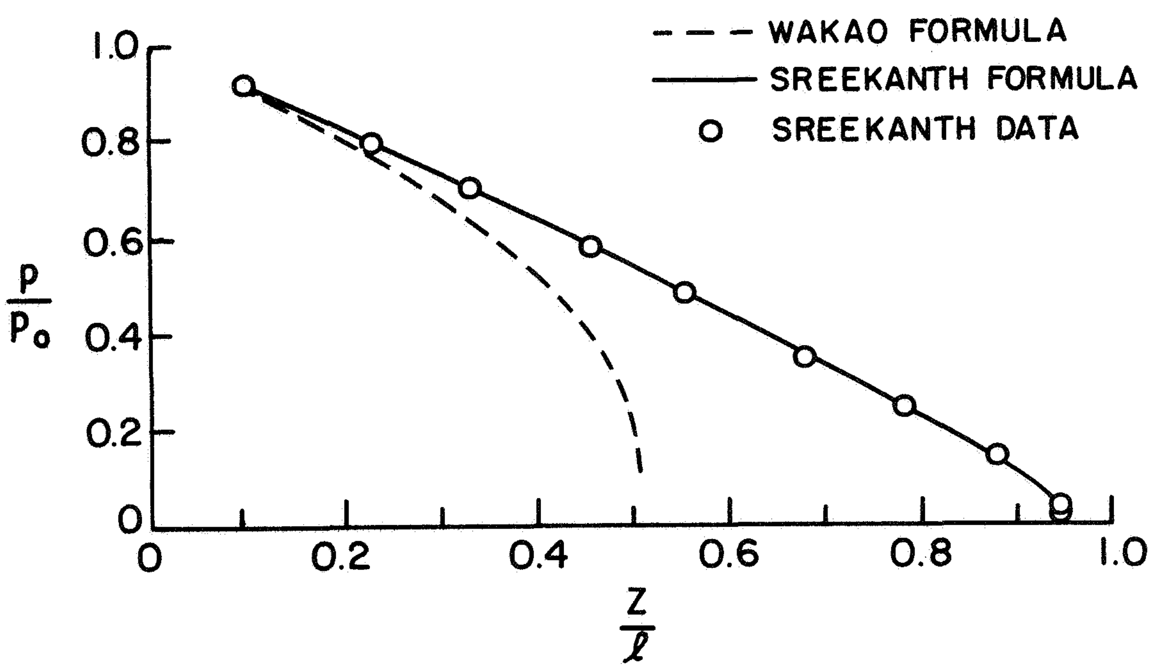


FIGURE 1-4 TYPICAL COMPARISON OF WAKAO AND SREEKANTH FORMULAS

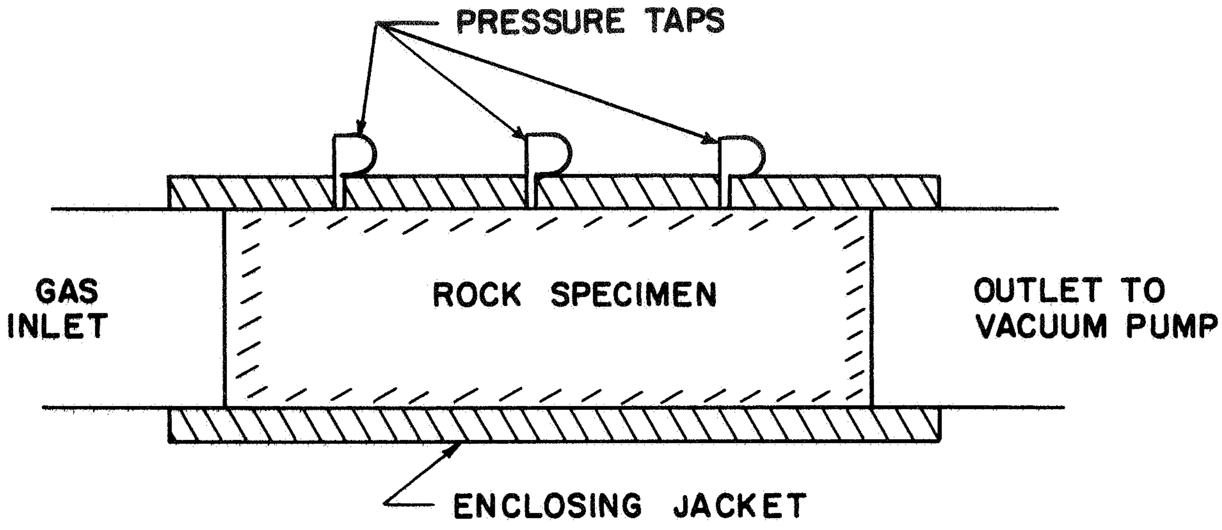
V. PROPOSED EXPERIMENTATION

In the final quarter of this year's work, plans were made for a series of experiments aimed at developing a probe prototype. To simulate the effects of the lunar environment, all experiments will be conducted on rock samples enclosed in a vacuum chamber. To our knowledge, this will be the first work of this kind that has been attempted.

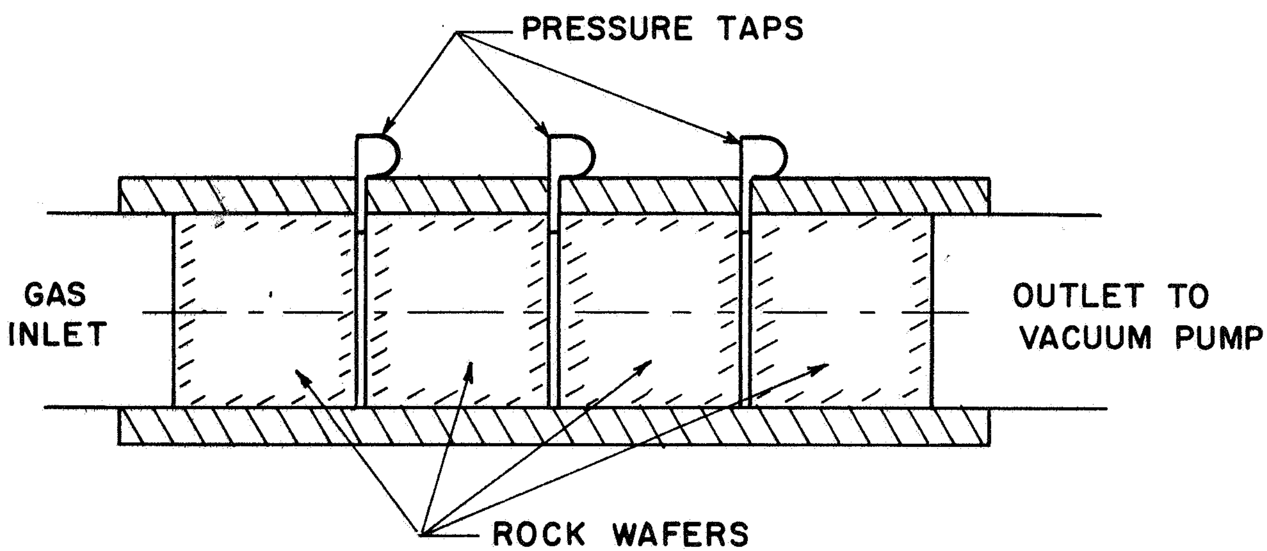
Facilities currently available in the Aeronautical Sciences Laboratories are easily able to maintain significant flow volumes at a vacuum of 10^{-6} Torr , and with the aid of auxiliary equipment, a vacuum of 10^{-8} Torr can be reached. These pressure levels will enable us to examine the effects of rarefied gas flow without having to duplicate the much lower vacuum (10^{-14} Torr) of the lunar environment.

The first experiment will consider one-dimensional flow. This is the simplest geometry, and will enable us to develop a familiarity with appropriate procedures for handling gas flow through porous media under high vacuum conditions. Two basic rock configurations are being considered. The first consists of a single, cylindrical rock specimen, with pressure taps embedded in the rock, cf. Figure 1-5a. In the second configuration, the pressure taps are placed in gaps between cylindrical "wafers" of rock, cf. Figure 1-5b, in order to insure that measurements of average pressure are obtained.

By selecting rocks of appropriate permeability and controlling absolute pressures at the necessary levels, it should be possible to obtain conditions that vary from continuum viscous flow to flow with a high degree of



(a)



(b)

FIGURE 1-5 SCHEMATICS OF PROPOSED CONFIGURATIONS FOR EXPERIMENTS ON ONE-DIMENSIONAL FLOW

rarefaction. In so doing, the results obtained will provide a necessary test of the local similarity method.

One of the fundamental considerations in the design of the lunar permeability probe is its range of application. Over what range of permeabilities can the lunar probe be expected to yield satisfactory results, given the operating conditions that prevail on the lunar surface? Can the probe be applied to consolidated and/or unconsolidated rocks? These problems can be investigated by varying the permeability of rock samples, and noting those conditions of temperature, flow rate, and absolute pressure that are practical for the envisioned probe.

Another consideration is the type of gas to be used. Is there any advantage to be gained by using a single component gas rather than a gaseous mixture? From the experimental standpoint, it will be possible to use both approaches and thus provide data for another design factor of the probe.

In subsequent experiments, the applicability of the envisioned probe to spherical flow conditions will be investigated. This work will provide design criteria for such features of probe construction as the necessary size of the probe skirt, and the optimum positioning of pressure taps. The results of this work can then be used to build and test a probe prototype.

VI. CONCLUSIONS

A theory of gas flow in porous media appropriate to the lunar surface permeability probe has been developed. This theory is based on the concept of local similarity and utilizes the assumptions of steady, isothermal, and symmetric flow assumptions that have proven possible for terrestrial flows. This concept of local similarity is more general than the few

existing theories for transitional flow in porous media. It is entirely consistent with what is judged to be the best of such work. The design of experiments to check this theory and develop a probe has begun.

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APPENDIX A

AVERAGE QUANTITIES FOR A SYMMETRIC FLOW

In Equation (1-5), the area fraction α is defined

$$\alpha \equiv \frac{A_p r_1}{A\ (r_1)} \tag{A-1}$$

where

$$A(r_1) \equiv \begin{cases} A_r & \text{one dimensional} \\ 2\pi r_1 \ell c & \text{cylindrical symmetry} \\ 4\pi r_1^2 & \text{spherical symmetry} \end{cases} \tag{A-2}$$

Here r_1 is a reference length large compared to a typical pore dimension, and $A_p(r_1)$ is the total pore area normal to the mean flow at $r = r_1$. It is assumed that α is a constant property of the porous medium. This assumption requires that the rock be isotropic, and becomes more accurate with increasing values of r_1.

The "average" velocity is defined

$$\bar{v}\ (r) \equiv \frac{1}{A_p(r)} \iint_{A_p(r)} \underline{v}_p\ (r,y,z) \cdot \underline{dA}_p \tag{A-3}$$

Here $\underline{v}_p$ is the "local" velocity in an individual pore. Similarly, the mass flow rate can be expressed

$$Q = \iint_{A_p(r)} \rho_p(r,y,z)\ \underline{v}_p\ (r,y,z) \cdot \underline{dA}_p$$

$$\equiv \overline{\rho v}\ (r)\ A_p(r) \qquad \text{(A-4)}$$

where ρ_p is the local density in a pore. Now continuity equation can be expressed, cf. Equation (1-5),

$$\alpha\ \bar{\rho}\ (r)\bar{v}\ (r)r^j = \hat{Q} \qquad \text{(A-5)}$$

In order to satisfy Equation (A-5), the average density is then defined as

$$\bar{\rho}\ (r) \equiv \frac{\overline{\rho v}(r)}{\bar{v}(r)} \qquad \text{(A-6)}$$

In the test of this report, the bars have been dropped from all average quantities.

APPENDIX B*

KNUDSEN NUMBER

From the kinetic theory of gases,

$$\mu = \frac{1}{2} b_2 \lambda \rho \bar{V} \tag{B-1}$$

where b_2 is a dimensionless constant of order one, and λ is the mean free path. Invoking the perfect gas law, taking $b_2 = 1$, and rearranging Equation (B-1),

$$\lambda = \frac{2\mu RT}{\bar{V}} \frac{1}{p} \tag{B-2}$$

A Knudsen number based on L can be defined, $Kn_L \equiv \frac{\lambda}{L}$. Then

$$Kn_L = \frac{2\mu RT}{L\bar{V}} \frac{1}{p} \tag{B-3}$$

so that, for an isothermal flow, Kn_L, and p are inversely proportional.

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*As a general reference, see Chapman, S. and T. O. Cowling (1964), The Mathematical Theory of Non-Uniform Gases, Cambridge Univ. Press, New York.

SYMBOLS

a	source radius
a_1	constant (cf. Eq. 13)
$A = \pi a^2$	area of fluid source
$A(r)$	reference area
A_p	total pore area normal to flow
A_r	total cross-sectional area of rock
b	constant [cf. Eq. (13)]
b_1	constant of order one appearing in slip boundary condition
b_2	constant of order one
c_1	constant [cf. Eq. (13)]
c_2	constant [cf. Eq. (13)]
C	master diffusion coefficient
$\tilde{C}$	dimensionless master diffusion coefficient
d	average pore diameter, and tube diameter
D	momentum diffusivity appropriate to a particular flow regime
$D_{KA} = \frac{1}{3} d\overline{V}$	Knudsen diffusivity
F	universal function of dimensionless pressure, equivalent to $\tilde{C}^{-1}$
j	index for one dimensional cylindrically symmetric, or spherically symmetry flow
k	continuum permeability
$Kn_d = \frac{\lambda}{d}$	local Knudsen number based on pore (or tube) diameter

$Kn_{d_o} = \frac{\lambda_o}{d}$ reference Knudsen number based on pore (or tube) diameter

$Kn_L = \frac{\lambda}{L}$ local Knudsen number based on L

ℓ tube length

ℓ_c reference length for cylindrically symmetric flow

L square root of generalized permeability

L^2 generalized permeability

p pressure

p_e background pressure

p_o reference pressure

Q mass flow rate

$\hat{Q}$ reference mass flow rate for symmetric flow

r length coordinate for symmetric flow

r_1 reference length

R gas constant

$Re = \frac{4}{\pi} \frac{Q}{\mu d}$ Reynolds number based on mass flow rate

t time

T temperature

v gas velocity

v_p local velocity in pore

$\bar{V} = \sqrt{\frac{8RT}{\pi}}$ mean thermal speed of gas

$\dot{V}$ volumetric flow rate

z axial coordinate along tube

α area fraction

$\zeta = \frac{L\bar{V}}{2\mu RT}\, p = \frac{1}{Kn_L}$ dimensionless pressure

λ mean free path

λ_o mean free path at reference conditions

μ viscosity

ρ density

ρ_p local density in pore

$\tau = \frac{4Dt}{a^2}$ dimensionless time

ϕ porosity

χ weakly varying function of Kn_d, assumed a constant of order one